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Well Drawdown in Unconfined Aquifiers Under Non-Steady Conditions

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COMPUTER METHODS FOR TRANSIENT ANALYSIS OF WATER-TABLE AQUIFERS

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Abstract

The drawdown in a pumped unconfined aquifer can be analyzed by computer methods. The computer can give simultaneous solutions for flow in the saturated and unsaturated zone. The surface of seepage can also be included in the analysis.

Introduction

Most analyses of drawdown around pumped gravity wells have been concerned with steadystate flow with horizontal replenishment and with or without vertical replenishment. The familiar Dupuit-Forchheimer assumptions are used as the basis of the resulting solution. The rate of discharge from the wells is given quite accurately, but the distribution of hydraulic head, especially in the vicinity of the well, is not given accurately (Peterson, 1957). The surface of seepage at the well is ignored in the Dupuit-Forchheimer approach.

Other more exact steady-state analyses include the work of Kirkham (1964), Yang (1949), and Hall (1955), who used exact mathematical analysis, numerical techniques, and tank models.

Transient conditions or nonsteady-state conditions have been analyzed by solutions of Boussinesq's equation written in cylindrical coordinates. These solutions all suffer from the limitations imposed by the Dupuit-Forchheimer assumptions: The seepage surface is ignored, and the hydraulic gradient at any point is assumed to be equal to the slope of the piezometric surface above the point. Theis (1935) and Boulton (1954) have reported in some detail the usefulness and limitation of these equations, whereas Wenzel (1942) has demonstrated their value in certain cases for determining formation constants from field pumping tests. Measurements made close to the well can lead to serious errors in the application of the formulas.

Hantush and Jacob (1955) derived equations to express drawdown from nonsteady radial flow in an infinite leaky aquifer (i.e., sand bed on top of aquifer). When leakage is very small or when the pumping time is sufficiently small so that leakage does not enter the main flow, the equations approximate to the Theis equation. Other investigators have obtained analytic expressions for inflow into wells, but the studies are principally concerned with flow in a confined aquifer or for a steady-state condition. Rather complete reports of these studies are given by Todd (1959) and others.

In analyzing drawdown for an unconfined aquifer, some important parameters to be incorporated in the study are the relationships among water content θ, the aquifer hydraulic conductivity K, and the capillary pressure head H of the unsaturated portion of the aquifer. When these relationships are considered in analytical analysis, the complexity of the analysis usually prevents their inclusion. With the aid of high-speed digital computers, these parameters can easily be incorporated into drawdown studies that yield various pumping times the free water surface, hydraulic head, and water contents in the flow region.

The purpose of this paper is to present methods for utilizing numerical analysis and computer operations to solve problems of drawdown around a pumped well in an unconfined aquifer. Over-all, the procedure is that of solving the basic flow equations for specified boundary conditions. The method takes into account the properties of the unsaturated portion of the aquifer and the contribution of vertical flow. Although this report deals only with flow into wells, the procedure is applicable to a variety of water flow problems.

Basic Equations

Equations 1 and 2 are the basic flow equations used in the study. Because of the well geometry, it is convenient to express them in cylindrical coordinates for the case of angle symmetry. Equation 1 applies to flow in the saturated protion of a porous medium, whereas equation 2 is applicable to the unsaturated part. Both equations must be solved simultaneously when the free surface (water table) is either receding or rising.

$$\frac{K_0}{r}\frac{\partial \phi}{\partial r} + \frac{\partial}{\partial r} \quad K_0\frac{\partial \phi}{\partial r} \quad + \frac{\partial}{\partial z} \quad K_0\frac{\partial \phi}{\partial z} \doteq 0 \tag{1}$$

$$\frac{K}{r}\frac{\partial \phi}{\partial r} + \frac{\partial}{\partial r} \quad K\frac{\partial \phi}{\partial r} \quad + \frac{\partial}{\partial z} \quad K\frac{\partial \phi}{\partial z} = \frac{\partial \phi}{\partial t} \tag{2}$$

where

ϕ $(P/pg) + z$ is the hydraulic head;

P is the pressure;

$H =$ $(-P/pg) \doteq z - \varphi$ is the capillary pressure head, herein defined only for negative values of P;

K_0, K represent the water conductivity in the saturated and unsaturated portions of the porous medium, respectively. For a uniform porous medium, K_0 is considered constant for all positive values of P, whereas K is a function of the capillary pressure head H. As used herein, K_0 and K will be referred to as the saturated and capillary conductivity, respectively. Either K_0 or K may differ in the r and z directions if the medium is anisotropic;

θ is the water content of the medium expressed as a volume fraction. Like the conductivity, it is assumed of constant magnitude θ_0 for all positive values of P and a function of the capillary pressure head H for negative values of P;

r, z are the coordinate directions, z being parallel to the earth's gravitational field;

p, g are the fluid density and gravitational field strength, respectively;

t is time.

In unsaturated porous media, the relationship among K, θ, and H depends on the size and arrangement of the solid soil particles as well as antecedent conditions of drainage or water replenishment hysteresis. In this study we shall deal only with drainage from an initially saturated medium. It will further be assumed that water content and conductivity are uniquely determined by the capillary pressure head. The relationships used to calculate K and θ are given by equations 3 and 4. The graphical representation of these equations is shown in Figure 1 for several values of the constant A. The curves for larger values of A are considered typical of the conductivity in actual media. The values of A used in calculating K and θ for specific examples are given later in the paper.

$$K = K_0/(AH^3 + 1) \tag{3}$$

$$\theta = \theta_0/(AH^3 + 1) \tag{4}$$

where A is a constant.

To utilize numerical analysis procedures and computer operations, it is necessary to express equations 1 and 2 in finite difference form. A modified Gauss-Sidel iterative method is used (Forsythe, 1956) to obtain a system of nonlinear equations. The procedure is essentially that used by Reisenaur (1963) and others in which estimates of the first derivative are based on central difference. The second derivatives

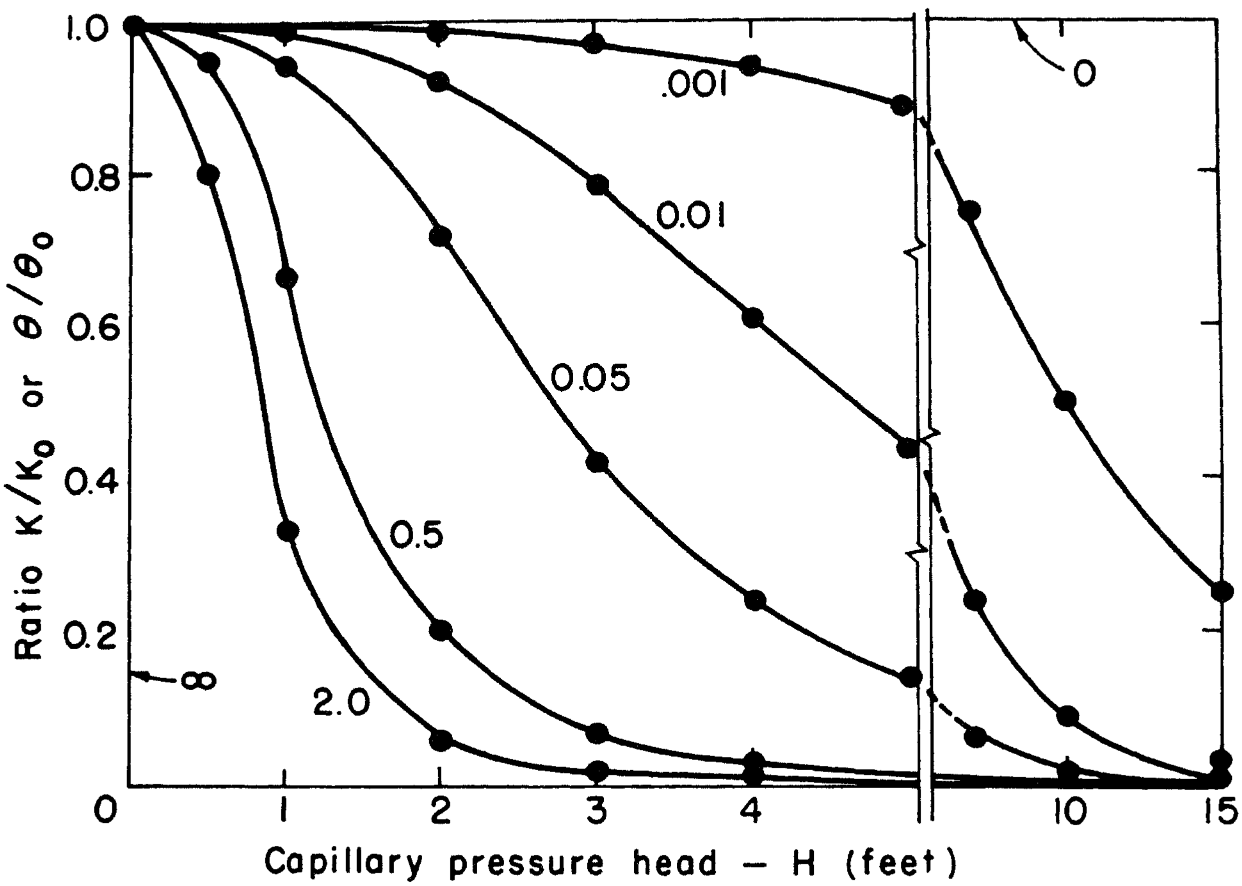

FIGURE 1. RELATIONSHIP AMONG THE CAPILLARY PRESSURE HEAD AND CAPILLARY CONDUCTIVITY ON MOISTURE CONTENT.

are obtained by combining 'backwards and forward' differences. The finite difference expression for equation 2, for example, is given by equation 5. In the latter expression the letters a, b, c, and d refer to the mesh dimensions shown in Figure 2, and the i, j matrix system corresponds to the one shown in the same graph. In following the procedure described above, one derives the first term in equation 5 from the first term in equation 2, the second and third terms in 5 from the second term in 2, and the fourth term in 5 for the third term in 2. The term on the right-hand side of equation 5 is derived from the similarly located term in equation 2.

The finite difference form of equation 1 is similar to equation 5. It will differ from equation 5 in that (a) the capillary conductivity K in equation 5 is replaced by the saturated conductivity K_0 and (b) the right side of equation 5 is set equal to zero. For a uniform porous medium (i.e., no stratification), K_0 is not a function of the coordinates (r, z), and the second and fourth terms in 5 are also set to zero

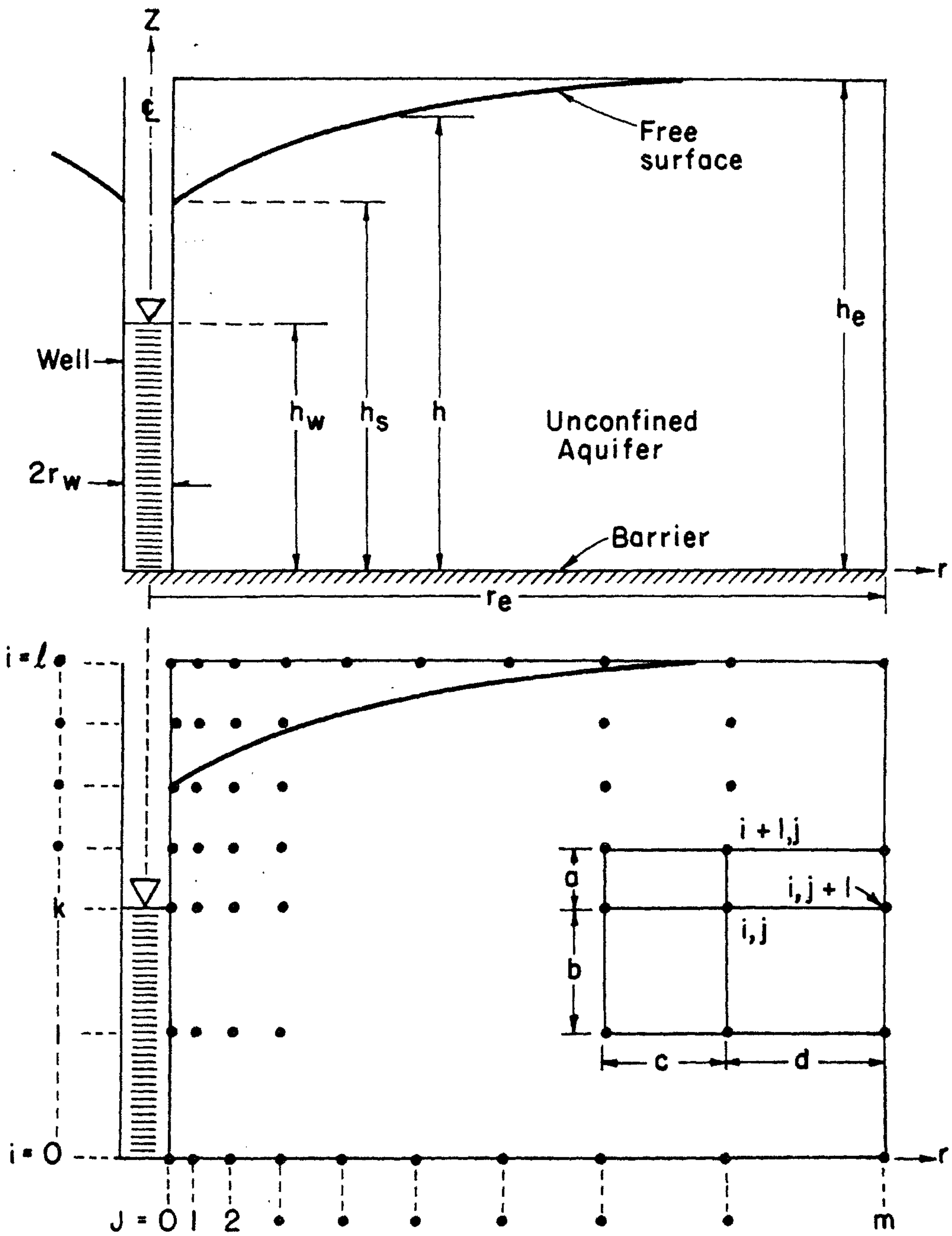

FIGURE 2. FINITE DIFFERENCE MESH (LOWER FIGURE) FOR WELL PROBLEM (UPPER FIGURE).

$$\frac{K_{i,j}}{r_{i,j}} \quad \frac{\phi_{i,j+1} - \phi_{i,j-1}}{(c+d)}$$

$$+ \frac{(\phi_{i,j+1} - \phi_{i,j-1})(K_{i,j+1} - K_{i,j-1})}{(c+d)^2}$$

$$+ \frac{2K_{i,j}}{(c+d)} \quad \frac{(\phi_{i,j+1} - \phi_{i,j})}{d} - \frac{(\phi_{i,j} - \phi_{i,j-1})}{c}$$

$$+ \frac{(\phi_{i+1,j} - \phi_{i-1,j})(K_{i,j} - K_{i-1,j})}{(a+b)^2}$$

$$+ \frac{2K_{i,j}}{(a+b)} \quad \frac{(\phi_{i+1,j} - \phi_{i,j})}{a} - \frac{(\phi_{i,j} - \phi_{i-1,j})}{b}$$

$$= \frac{(\Theta_{i,jn} - \Theta_{i,j})}{(t_n - t)} \qquad (5)$$

where

i,j are the index integers of the grid points in the z and r directions, respectively (see figure 2);

n is a subscript that indicates that a given quantity occurs at the time t_n. (The terms not containing the subscript n occur at the time t_{n-1});

K and Θ are defined by equations 3 and 4.

Computational Procedure

The general procedure will be described first and then followed by specific examples. The numerical analysis procedure and computer operations utilize an integer index system. This system can be made to correspond with an ij grid system such as shown in Figure 2. For each grid point ij, the following information is stored for immediate access in the computer memory: The potential ϕ, the capillary pressure P, the water content θ, and either the conductivity K_o or the capillary conductivity K. Each quantity is stored as a two-dimensional matrix and can be called sequentially from storage by suitable reference to the ij integer system. The radial distance r and the free surface elevation h at any vertical plane j are also stored as one-dimensional matrices. The elevation z at any horizontal plane i is stored similarly.

Initial condition. First, one assigns initial values of ϕ, P, θ, and K to all grid points. These values are chosen to simulate the hydrologic conditions both in the interior and along boundaries of the porous medium at time $t = 0$.

Step 1. Equation 5 is applied to all interior grid points (and sometimes boundary ones) where the hydraulic pressure $P \leqq 0$. From this equation, one calculates a change in water content $\Delta\theta_n$ when an arbitrary but small time interval Δt_n is used. The elapsed time is then $t_n = t_{n-1} + \Delta t_n$. (When Step 1 is initially applied, n is 1; t_{n-1} is zero and $t_n = \Delta t_n$.) Following each calculation, the new water content $\theta_n = \theta_{n-1} + \Delta\theta_n$ is computed, and this value replaces θ_{n-1} in computer storage. (For $n = 1, \theta_{n-1}$ is the moisture content assigned initially.) The capillary pressure H_n is now calculated from θ_n by utilizing equation 4. Following this, one computes an adjusted value of K_n by substituting H_n in equation 3. Similarly, by utilizing H_n, one calculates $P_n = H_n \rho g$ and $\theta_n = z - H_n$. Thus, at the end of Step 1, values of ϕ, P, θ, and K at specified grid points have been adjusted due to the application of equation 5. They represent the magnitudes of these parameters at the time t_n. These adjusted values are now stored in computer memory in place of those assigned initially.

Step 2. The finite-difference form of equation 1 is used to calculate the hydraulic head ϕ for all interior grid points (and some boundary points) where $P > 0$. The procedure for carrying out this step is that of repeatedly solving the appropriate finite difference equation, in a sequential manner, until only negligible changes occur in θ-values during any two successive traverses of the grid points. After each traverse, the most recently computed value of ϕ is stored in computer memory. At the termination of these traverses, the stored values of ϕ represent those existing at the time t_n. From these values, adjusted values of $P_n = z - \phi_n$ are calculated and stored in computer memory.

At this point, it may be noted that Step 2 has the over-all effect of adjusting the hydraulic heads in the saturated region to conform with any changes in ϕ and K in the unsaturated portion. At grid points adjacent to the free surface (i.e., $P = 0$), the magnitudes of ϕ and K in one region are influenced by values in the adjacent region. Thus, if Step 1 brings about a reduction in water contents and hydraulic heads, the effect of Step 2 is to also lower the hydraulic head in the saturated portion. This leads to a lowering of the free surface and an enlargement of the unsaturated region.

Step 3. If needed, adjustments are made at boundary grid points other than those altered in Steps 1 and 2. For example, if a constant pumping rate is to be simulated, one may adjust the water level elevation in the well to satisfy this condition. Since the hydraulic head at the well periphery is affected by such a change, the hydraulic head at these grid points must be altered accordingly. These changes in ϕ are also made in memory storage.

At the end of Step 3, all stored values ϕ, K, P, H, and Θ corresponds to aquifer conditions at the time t_n. To continue with the computer operation, one simply repeats Steps 1, 2, and 3. The time interval now becomes Δt_{n+1}, and the elapsed time is calculated by adding this increment to t_n. The water content change is $\Delta \Theta_{n+1}$. Thus, after Steps 1, 2, and 3 are applied the second time, all values of ϕ, K, P, H, and Θ correspond to the elapsed time t_{n+1}.

The entire operation is then repeated, sequentially, time after time, until a given pumping time has elapsed, the free surface has receded to a certain elevation, or until some other prescribed condition has been fulfilled. At any point in the operation, a computer printout of current values of the various parameters can be made.

Flow Problem Example - Gravity Well With Horizontal Replenishment

The steps followed in an analysis can be illustrated by the flow problem shown in Figure 2. An unconfined, uniform, and saturated aquifer of thickness h_e is completely penetrated by a single well of radius r_w. A steady-state pumping rate Q is initiated and consequently followed by a decline in the free water surface in both the aquifer and well. There is no replenishment across the ground surface, and fluid withdrawal is accompanied by air entry into the aquifer voids. The free surface at the radial distance r_e remains fixed at the ground surface. An equilibrium condition is eventually reached, during which the free surface becomes static and the pumping rate equals the radial inflow at the boundary r_e.

Flow in the aquifer is assumed to be Darcian in nature, and equations 1 and 2 are thus applicable. Below the free surface, the hydraulic conductivity is of constant magnitude K_o, whereas in the partially unsaturated region, the conductivity K is dependent on the capillary pressure head H as given by equation 3. For $P \geqq 0$, the water content θ_o is equal to some constant value, say 0.40, whereas for $P < 0$, θ is given by equation 4. The aquifer is isotropic with regard to its conductivity, both above and below the free surface. The impedance to flow at the well is assumed to be negligible.

The boundary conditions include those at the well, the upper and lower boundaries of the aquifer, and the vertical plane at r_e. For grid points at the well but below the free surface, hydraulic heads are assigned as follows:

$$\phi = h_w (0 \leq z \leq h_w) \tag{6A}$$

$$\phi = z (h_w \leq z \leq h_s) \tag{6B}$$

The latter conditions designates the surface of seepage.

Above the free surface, there is no radial flow at the well, and the term $\partial\phi/\partial r$ is equal to zero. For purpose of numerical analysis, this condition is met by imposing at the well center line a mirror image of the vertical plane $j = 1$. Thus, in using equation 5 for the column $j = 0$, the magnitudes of the various parameters at the grid point $(i, j - 1)$ are set equal to those at $(i, j + 1)$, and the mesh dimension c is set equal to d. At the upper and lower boundaries of the aquifer, there is no vertical flow, and the quantity $\partial\phi/\partial z$ is equal to zero. These boundary conditions are met by imposing at the ground surface the mirror image of the plane at $i = 1$ and at the lower aquifer boundary the mirror image of the plane at $i = 1$. The grid points along the boundary at r_e are assumed of constant hydraulic head h_e.

Initial conditions: At time zero, the water level in the well is set at some value h_w, whereas the free surface in the aquifer is fixed at the ground surface. The value of h_w is chosen so that it will bring about an inflow into the well approximately equal to a designed pumping rate Q. These initial conditions are met in the computational analysis by first assigning values of $\phi = h_e$ and $K = 1$ to all grid points along the ground surface and at the plane r_e. The parameters at the latter boundary remain unchanged in all subsequent steps. Values of $P = 0$ are also assigned to points along the ground surface and $P = h_e - z$ to points at the distance r_e. The appropriate values of ϕ are also assigned to grid points at the well (Equation 6). Arbitrary values of ϕ are assigned to all other grid points, whereas K, P, and ϕ are initially set at 1, 0, and 0.4, respectively. Then while retaining the original values of ϕ at the well, the ground surface, and at r_e, a steady-state analysis is attained by repeatedly traversing all other grid points and solving the finite difference form of equation 1. Adjusted values of K, ϕ, P, and θ are then stored in computer memory.

Phase 1. Water content changes are now calculated with equation 5. In the initial step, equation 5 applies only to grid points along the ground surface ($i = 1$). Hereafter, this operation applies to all points where $P \leq 0$, except those along the well periphery. At this boundary, equation 5 does not apply to points whose elevation is less than h_s. Following the application of equation 5, adjustments are made in the stored values of ϕ, K, P, and θ as described in Step 1.

Phase 2. A steady-state analysis is attained by carrying out the operation given by Step 2. This operation applies to all grid points where $P > 0$, save those at the well. The most current values of ϕ and P are stored in computer memory.

Phase 3. Adjustments are now made in the water level h_w, and the free surface elevation is calculated. The inflow into the well Q' is calculated and compared with the prescribed pumping rate Q. If they differ by more than, say 1 percent, the water level in the well is adjusted upwards or downward a small amount Δh_w. The potential at points along the well is also adjusted in accordance with equation 6. The quantity Q' is determined by summing the instantaneous flow passing through the cylindrical shell bounded by the vertical planes $j = 0$ and 1. For every vertical plane r_j $(j > 0)$, the free surface elevation h is calculated by first locating the adjacent, vertical grid points at which P changes from positive to negative values. The pressure is expressed as a linear function of elevation z in this interval. Then extrapolation of P is made to obtain the elevation where P is zero. The free surface at the well h_s is now determined by first expressing h as a linear function of r for distances near the well (i.e., at $j = 1$ and 2). Then h, is determined by extrapolating h to r_w.

The computations now repeat Phases 1, 2, and 3 in a sequential manner. That is, water content changes are made in the unsaturated zone, a steady-state analysis is made in the saturated region, and then adjustments are made in the water level in the well. A printout can be made at any point in the operations, an example of which is shown in Figure 3.

<u>Other Computational Considerations</u>

A variable time increment Δt is used to reduce computer running time. The initial value is of low magnitude, say .001 K_0, and is then slowly increased in subsequent steps. This permits suitable adjustments to be made in h_w before an appreciable pumping time has elapsed. Thereafter, each successive time increment is increased above its former

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TOTAL NO. OF ITERATIONS= 288 OVER RELAXATION CONSTANT= 1.6000

TOTAL TIME ELAPSED= 9.02339 DAYS

LAST TIME INCREMENT = 0.503 DAYS

CURRENT VALUE RESIDUALS = 0.18 FEET

PRESCRIBED PUMPING RATE = 5000. C.F.D.

ACTUAL PUMPING RATE = 4945. C.F.D.

HEIGHT WATER IN WELL = 24.00 FT.

Well

POTENTIALS AT GRID POINTS - FT.

92.76	93.19	93.68	94.54	95.63	97.47	100.00	Ground surface
85.40	88.51	90.48	92.93	95.06	97.42	100.00	
80.00	85.01	88.34	91.90	94.69	97.38	100.00	
40.00	65.55	78.19	87.74	93.31	97.23	100.00	
24.00	57.41	73.87	85.99	92.76	97.16	100.00	Barrier

HYDROSTATIC PRESSURE - FT.

-7.24	-6.81	-6.32	-5.46	-4.37	-2.53	-0.00	
-4.60	-1.49	0.49	2.93	5.06	7.42	10.00	Saturated zone
0.00	5.01	8.33	11.89	14.69	17.38	20.00	
0.00	25.54	38.20	47.73	53.31	57.23	60.00	
24.00	57.40	73.85	85.98	92.75	97.16	100.00	

HYDRAULIC CONDUCTIVITY - FT. PER DAY

0.05	0.06	0.07	0.11	0.19	0.55	1.00
0.17	0.86	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00

WATER CONTENT

0.29	0.30	0.32	0.34	0.37	0.39	0.40
0.36	0.40	0.40	0.40	0.40	0.40	0.40
0.40	0.40	0.40	0.40	0.40	0.40	0.40
0.40	0.40	0.40	0.40	0.40	0.40	0.40
0.40	0.40	0.40	0.40	0.40	0.40	0.40

RADIAL DISTANCES FROM WELL CENTERLINE - FT.

1.00	10.00	25.00	55.00	100.00	200.00	400.00

WATER TABLE ELEVATIONS - FT.

85.89	87.75	90.84	93.55	95.39	97.46	100.00

FIGURE 3. PRINTOUT FOR SAMPLE PROBLEM.

value by a factor, usually 1.05 to 1.10, until an upper limit is reached. The latter is based on the maximum permissible change in ϕ and K during any time increment. If this limit is exceeded, the subsequent time increment is reduced by a factor, say 0.95.

It is necessary to use a small mesh size near the well, particularly in the unsaturated portion of the aquifer. An additional factor that influences mesh size is the spatial change in K and θ as a result of unsaturation. A close mesh is required when there are marked changes in these quantities. If the relationships among K, θ, and H are given by the left-most curves in Figure 1, a much smaller mesh will be needed than for those curves at the right.

The grid pattern illustrated in Figure 2 had been used satisfactorily in these studies with a minimum of computer running time. A small vertical mesh size a is used in the unsaturated region, and a larger size b is employed elsewhere. One must anticipate an elevation ($i = k$ in graph) below which there will be continual saturation. A ratio b/a as large as 4 or 6 has been used. A radial mesh size has been employed that increases exponentially with distance from the well. The mesh size at the well should be approximately equal to a with only small increases until the 3rd and 4th grid point is exceeded. Values of a as large as 5 feet can be used if K and θ are given by the uppermost curve shown in Figure 1 ($A = .001$), whereas values only one-tenth this magnitude could be tolerated for the lower curve ($A = 2.0$). In the final analysis, one will find it convenient to analyze a flow problem with increasingly smaller mesh values. When the results do not vary significantly between successive runs, the larger magnitude of a is usually selected to reduce computer running time. For the curve shown in Figure 1 with A = ∞, (i.e. no flow in the unsaturated region) the procedure described in this report cannot be used.

In utilizing equation 5 to calculate water content changes (see Step 1), an implicit assumption is made that the quantities ϕ, K, and θ do not change during the chosen time interval Δt. This is not true, and the greatest deviation usually occurs in K. When the spatial change in K is large, this assumption may lead to difficulties. It can be alleviated by utilizing the previous change in K to predict the subsequent value. From the current and predicted value of K over the time interval Δt, an average value of K can be calculated and used.

Illustrative Flow Cases

Equipotentials and free surface positions are shown in Figure 4 for a fully penetrating well. The upper graph results from a computer analysis after 0.06 days of pumping, whereas the lower graph represents equilibrium conditions as determined by computer and by sand tank studies of Hall (1955, Figure 14). Hall used a 15-degree sector of the flow region. In both tank and computer analysis, water is maintained along the outer radial boundary to give an equipotential surface. In the tank study, a constant water level is maintained in the well until a steady-state flow rate into the well Q_1 is attained. The free surface

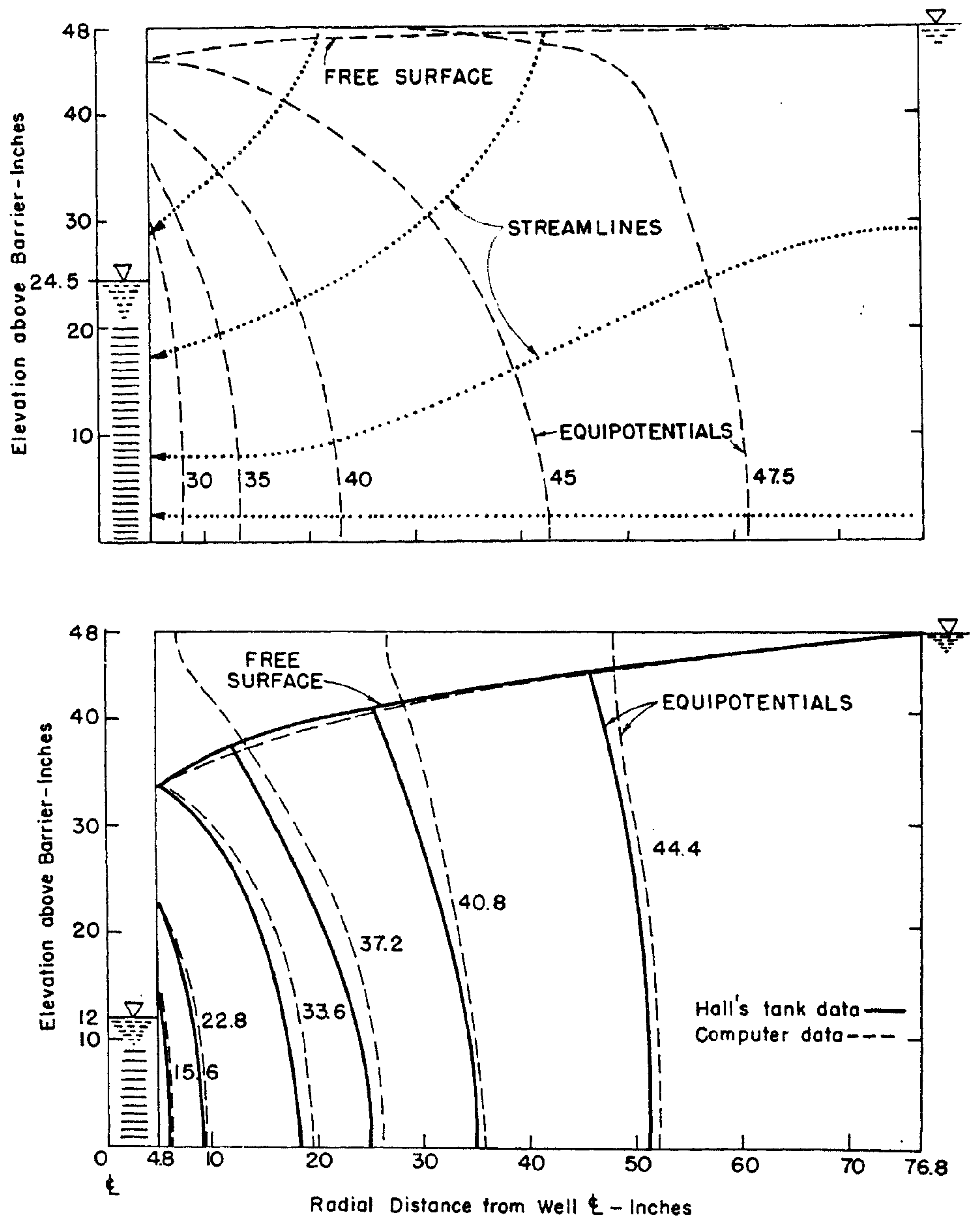

FIGURE 4. COMPARISON OF COMPUTER SOLUTION WITH HALL'S EXPERIMENTAL DATA.

and the potentials in the saturated zone were determined by utilizing piezometers installed in the tank walls.

In the computer analysis, a constant pumping rate $Q_1 = 2450\ K_0$ is maintained by adjusting the water level in the well. The aquifer is assumed saturated at time zero, and the free surface recedes to some equilibrium position after a long pumping time. Hydraulic heads and free surface elevations are obtained from a printout such as shown in Figure 3. The experimental value of the saturated conductivity was used for K_0. The relationship among K, θ, and H was not reported for Hall's sand, and an assumption was made that equations 3 and 4 were applicable. When the constant A in these equations was varied between 2 and 20 ft^{-3} (see Figure 1), no significant change could be noted in the equipotentials and free surface positions at equilibrium. Apparently this is due to the relatively small unsaturated zone and its location in a region where the hydraulic gradients are small. For the analysis reported herein, the magnitude of A in both equations 3 and 4 was 15. The mesh dimensions a and b were 3 and 8 inches, respectively, and the plane i = k was at 24 inches (see Figure 2). Radial grid points were at 4.8, 6.8, 9.8, 14.3, 20.3, 30.3, 46.3, and 76.8 inches. The maximum permissible change in H or ϕ during any time increment Δt was 0.04 in. The initial time increment was .001 K_0, and this parameter had increased to .55 K_0 when no subsequent change could be noted in the free surface elevation. The total computer time use was 4.6 minutes, of which 4.0 minutes was 'execution time.' Only a small fraction of the total printout data is utilized in this graph. At equilibrium the water level in the well and the free surface elevations did not differ more than 2 percent between experimental and computer-derived values.

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